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An analytical model for reinforced concrete beams with bolted side plates accounting for longitudinal and transverse partial interaction

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Abstract

The behaviour of bolted side-plated reinforced concrete (RC) beams is investigated in terms of the new concepts of the degrees of longitudinal and transverse of partial interaction. Composite bolted side-plated RC beams are unusual in that they exhibit not only longitudinal partial interaction, that is longitudinal slip at the steel/concrete interface, but that they also exhibit vertical or transverse partial interaction, which is slip transverse to the length of the beam. This paper derives mathematical models for the investigation of transverse partial interaction and its interaction with longitudinal partial interaction, and which are further developed to determine the distribution of slip strain, slip and the neutral axis separation of the steel and concrete components in terms of the degrees of transverse and longitudinal interaction. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Composite construction; Longitudinal response; Partial interaction; Plated beams; Slip; Slip strain; Transverse response

1. Introduction

This paper is concerned with the bolting of steel plates to the sides of reinforced concrete (RC) beams, which requires analysis techniques not only for the familiar longitudinal partial interaction in composite beams, but also for partial interaction transverse to the RC beam, and the interaction between this longitudinal and transverse partial interaction.

For the past two decades or so, bolting and adhesive bonding steel plates to the tension face of RC beams and slabs in order to strengthen and stiffen them has become common practice in the process of retrofitting and repairing building infrastructure (Jones et al., 1988; Oehlers, 1992), and recent work has focussed on the use of composite laminates in lieu of steel. Tension face attachments can lead to potential ductility problems caused by over-reinforcing, and so bolting steel plates to the sides of existing RC beams

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Nomenclature

E	Young's modulus
EI	flexural rigidity
F	half magnitude of applied load at midspan
h_c	distance between centroid of RC element and bolt level
h_e	distance between centroidal axes of plate and RC components
h_{na}	neutral axis separation of component strain profiles
h_p	depth of plate; distance between centroid of plate and bolt level
I	second moment of area
K_{si}	shear stiffness of bolt connection
K_1, K_2	constants
K_3	neutral axis separation constant
L	half span of beam
L_p	half span of plate
M_{app}	applied moment
M_{RC}	moment at centroidal axis of RC component
M_p	moment at centroidal axis of plate components
$(P_{sh})_x$	strength of shear connection in shear span x
q_{sh}	shear flow strength of shear connection or connector strength per unit length of beam
s_L	longitudinal slip
s_v	transverse or vertical slip
t_p	plate thickness
V	vertical shear force
x	distance from support to cross-section considered
ε	strain profile
ε_c, ε_p	strain in concrete and plate components respectively
ϕ_c, ϕ_p	curvature in concrete and plate components respectively
$\Delta\phi$	difference in curvatures
φ_L	degree of longitudinal partial interaction
φ_v	degree of transverse (or vertical) partial interaction

Subscripts

L	longitudinal
na	neutral axis
p	plate
RC	reinforced concrete
v	transverse (or vertical)
vfi	transverse full interaction
vpi	transverse partial interaction
x	section at distance x from support

enhances flexural strength and stiffness which can be achieved without any loss of ductility particularly when the plate is extended into the compression region of the beam.

The work reported in this paper is a continuation of recent studies (Oehlers et al. 1997; Nguyen et al., 1998) that pertain to the development of procedures for the analysis of side-bolted steel plates. These bolted

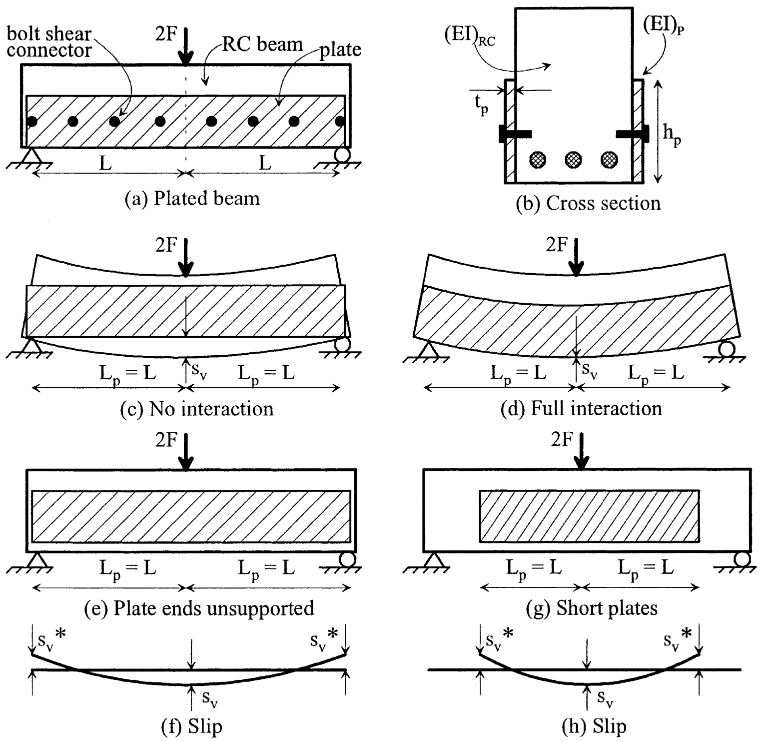

Fig. 1. Transverse interaction.

side-plated RC beams, such as that shown in Fig. 1, exhibit not only longitudinal slip at the steel/concrete interface (i.e. longitudinal partial interaction), but also vertical or transverse slip between the plates and the concrete beam (i.e. transverse partial interaction).

The aim of the current paper is to develop firstly the governing equations of this concept of transverse partial interaction, which can be used as a cornerstone for future nonlinear research on of this area of mechanics, and secondly to show how these equations can be used to develop design rules that incorporate transverse partial interaction. Methods for determining the transverse forces in composite bolted side-plated RC beams are developed first, and these are then used to quantify the distribution of slip strain, slip and neutral axis separation of the steel and concrete components in terms of the degrees of transverse and longitudinal interaction.

2. Transverse partial interaction

Transverse partial interaction affects both the strength and stiffness of a composite member. Consider, for example, the composite doubly-plated beam shown in Fig. 1(a) that is constructed with the soffits of the plates and beam in line, as shown in Fig. 1(b), where both the RC beam and plates are simply supported at their ends. It will be assumed that the shear stiffness of the bolt shear connection is K_{si}.

As the applied load $2F$ in Fig. 1(a) is gradually increased, the vertical deformation of the plate relative to the RC beam, i.e. the vertical slip s_v, depends on the shear stiffness of the shear connection K_{si}. When $K_{si} = 0$ (when there are no bolt shear connectors), then the applied load $2F$ only deflects the RC beam as

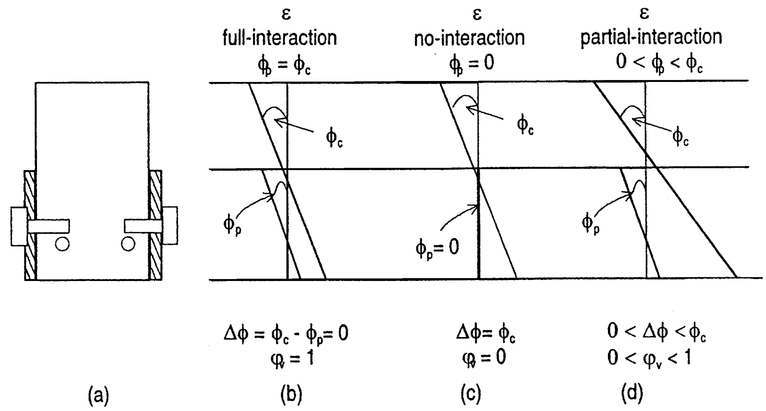

Fig. 2. Variation in strength and stiffness due to transverse partial interaction.

shown in Fig. 1(c). In this case there is no transverse interaction, and it will be denoted by a degree of transverse partial interaction $\varphi_v = 0$. On the other hand, as $K_{si} \rightarrow \infty$ which occurs when the plate is glued to the RC beam, then there is no relative vertical movement between the plate and beam as shown in Fig. 1(d). This condition is one of full transverse interaction, and could be defined by a degree of transverse partial interaction of $\varphi_v = 1$. Bolt shear connectors in composite plated beams of course resemble conventional stud shear connectors in composite steel and concrete beams, and so in reality $0 < K_{si} < \infty$ and therefore $0 < \varphi_v < 1$.

From a comparison of Fig. 1(c) and (d), it is clear that $\varphi_v = f(s_v)$. Since the vertical slip s_v can be derived by twice integrating the difference between the curvatures in the plate and RC beam $\Delta\phi$, it can be seen that $\varphi_v = f(\Delta\phi)$. This is illustrated in Fig. 2, which shows the strain profiles ε in a beam that exhibits longitudinal partial interaction (i.e. longitudinal slip) as well as transverse partial interaction. When there is transverse full interaction, that is $K_{si} \rightarrow \infty$, it can be seen in Fig. 1(d) that $s_v = 0$ and so the distributions of curvatures in the plate ϕ_p and in the beam ϕ_c are the same as shown in Fig. 2(b). Because of this, $\varphi_v = 1$ when $\Delta\phi = 0$. In contrast, when $K_{si} = 0$ in Fig. 1(c), then the plate is not deformed so that $\phi_p = 0$ as shown in Fig. 2(c). It may therefore be concluded that $\varphi_v = 0$ when $\Delta\phi = \phi_c$. Partial interaction occurs between these two extremes, as shown in Fig. 2(d).

It can also be seen from Fig. 2(c) that when $\varphi_v = 0$, the plate is unstrained and so the composite plated beam is at its weakest strength and lowest stiffness. However, when $\varphi_v = 1$ in Fig. 2(b), then the plate has its maximum strains in comparison to those in the concrete, and so the plated beam is at its strongest and stiffest. The partial interaction strength and stiffness will lie between these two extremes and, therefore, depend on the degree of vertical interaction φ_v as illustrated in Fig. 2(d). This paper will develop equations for describing transverse partial interaction in terms of $\Delta\phi$ (and hence φ_v), so that the degree of transverse partial interaction can eventually be used to quantify the flexural strength and stiffness of composite bolted side-plated RC beams.

3. Internal moments

Consider a simply supported composite plated beam in which a point load of magnitude $2F$ is applied to the concrete component at midspan, and where both the RC beam and the side plates are simply supported at their ends, as shown in Fig. 1(a). Furthermore, it will be assumed that there is a single line of shear connectors in line with the centroidal axis of the plate. Previous research on the fracture of shear connectors in composite beams (Oehlers and Sved, 1995) has shown that a convenient method for modelling the behaviour of the whole composite beam is to assume that both the concrete and steel components remain elastic, while all the shear connectors are plastic, that is they are fully loaded. In this paper, we will use this mixed analysis approach to develop mathematical models for side-plated beams. It will also be assumed in this model that the bolt shear connectors transfer the shear by dowel action as do stud shear connectors in composite beams and, hence, any beneficial effect of friction will be ignored.

The model that will be developed in this paper will be for the condition that the plate is vertically restrained by the supports of the beam, that is the half length of the plate L_p is equal to half the span of the beam L as shown in Fig. 1(c), so that the slip between the plate and the RC beam is s_v as shown. However, it may be worth noting that this model can be adapted to allow for different boundary conditions. For example the plate may be terminated at the supports as shown in Fig. 1(e) but not restrained vertically by the supports so that $L_p = L$ and the total change in slip that has to be accommodated by the transverse composite action is now $s_v + s_v^*$ as shown in Fig. 1(f). Furthermore, the plate may be stopped short of the supports as in Fig. 1(g) so that the change in slip $s_v + s_v^*$ has now to be accommodated over a shorter length of $L_p < L$, and the model can be applied to continuous beams where the points of contraflexure are represented by the supports of the simply supported beam.

A shear span of the plated beam of length x is shown in Fig. 3(a). Both plates in Fig. 1(b) are combined so as to give an equivalent single side plate of thickness $2t_p$ as in Fig. 3(b), and the strength of the shear connectors in this shear span of length x is denoted as $(P_{sh})_x$. There is thus an axial tensile force of $(P_{sh})_x$ in the plate component, and from equilibrium an axial compressive force in the beam component of the same magnitude, on this shear span. These forces are shown in Fig. 3(a), where they act at the centroids of the components which are at a distance h_e apart.

The moment acting at the centroid of the equivalent plate component of thickness $2t_p$ is denoted as $(M_p)_x$ as shown in Fig. 3(a), and the moment acting at the centroid of the RC component is denoted as $(M_{RC})_x$, where the centroid of the transformed RC beam does not obviously lie in the same place as the neutral axis. Further, let us define the applied moment acting at the end of the shear span of length x as $(M_{app})_x$. Because of longitudinal partial interaction between the plate and concrete components, their neutral axes do not coincide, and thus there is a slip strain between the strain profiles ε_c and ε_p as shown in Fig. 3(c).

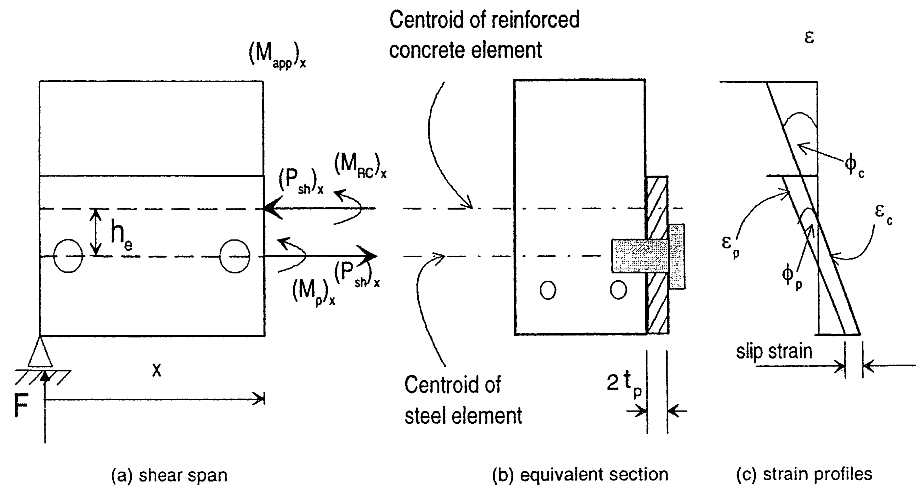

Fig. 3. Elastic–plastic analysis for simply supported side-plated bolted RC beam.

Consideration of the equilibrium of the moments shown in Fig. 3(a) produces

$$\left(M_{\mathrm{app}}\right)_x = \left(M_{\mathrm{RC}}\right)_x + \left(M_{\mathrm{p}}\right)_x + \left(P_{\mathrm{sh}}\right)_x h_{\mathrm{e}} \tag{1}$$

Consider now the beam in Fig. 1 when there is both longitudinal and transverse partial interaction between the plate and concrete components, as shown in Fig. 2(d). In this case, the curvatures of the components ϕ_{c} and ϕ_{p} shown in Fig. 3(c) are not equal, and the difference between these curvatures is $\Delta\phi$, so that

$$\Delta\phi = \phi_{\mathrm{c}} - \phi_{\mathrm{p}} = \frac{\left(M_{\mathrm{RC,vpi}}\right)_x}{(EI)_{\mathrm{RC}}} - \frac{\left(M_{\mathrm{p,vpi}}\right)_x}{(EI)_{\mathrm{p}}} \tag{2}$$

where $(EI)_{\mathrm{RC}}$ and $(EI)_{\mathrm{p}}$ are the flexural rigidities of the concrete and plate components respectively, and the subscript vpi denotes vertical (or transverse) partial interaction. The following internal moments, acting at the centroids of the concrete and plate components when there is transverse partial interaction, can be derived from Eqs. (1) and (2) in terms of $\Delta\phi$ as

$$\left(M_{\mathrm{RC,vpi}}\right)_x = \frac{m\left[(M_{\mathrm{app}})_x - (P_{\mathrm{sh}})_x h_{\mathrm{e}}\right] + \Delta\phi(EI)_{\mathrm{RC}}}{m+1} \tag{3}$$

$$\left(M_{\mathrm{p,vpi}}\right)_x = \frac{\left(M_{\mathrm{app}}\right)_x - (P_{\mathrm{sh}})_x h_{\mathrm{e}} - \Delta\phi(EI)_{\mathrm{RC}}}{m+1} \tag{4}$$

where m is the ratio of the component flexural rigidities $(EI)_{\mathrm{RC}}/(EI)_{\mathrm{p}}$.

Eqs. (3) and (4) can be simplified further in terms of $\Delta\phi_{\max}$, which is the maximum difference between the component curvatures, q_{sh} which is the shear flow strength, and the distance along the span x as

$$\left(M_{\mathrm{RC,vpi}}\right)_x = \frac{[m(F - q_{\mathrm{sh}}h_{\mathrm{e}}) + \Delta\phi_{\max}(EI_{\mathrm{RC}})/L]x}{m+1} \tag{5}$$

$$\left(M_{\mathrm{p,vpi}}\right)_x = \frac{[F - q_{\mathrm{sh}}h_{\mathrm{e}} - \Delta\phi_{\max}(EI_{\mathrm{RC}})/L]x}{m+1} \tag{6}$$

Furthermore, it has been shown (Oehlers et al., 1997) that the relationship between $\Delta\phi_{\max}$ and the degree of transverse partial interaction φ_{v} is given by

$$\Delta\phi_{\max} = \frac{L(1-\varphi_{\mathrm{v}})(F - q_{\mathrm{sh}}h_{\mathrm{e}})\sum\overline{EI}}{m+1} \tag{7}$$

where the degree of the transverse partial interaction φ_{v} is now defined as the ratio of the vertical shear force acting on the connectors due to partial transverse interaction V_{vpi} to the vertical shear force acting on the connectors when there is full transverse interaction V_{vfi}, that is $K_{\mathrm{si}} \to \infty$ and $\varphi_{\mathrm{v}} = V_{\mathrm{vpi}}/V_{\mathrm{vfi}}$, and where $\sum\overline{EI} = 1/(EI)_{\mathrm{RC}} + 1/(EI)_{\mathrm{p}}$.

Substituting Eq. (7) into Eqs. (5) and (6) and simplifying gives

$$\left(M_{\mathrm{RC,vpi}}\right)_x = (F - q_{\mathrm{sh}}h_{\mathrm{e}})\left(1 - \frac{\varphi_{\mathrm{v}}}{m+1}\right)x \tag{8}$$

$$\left(M_{\mathrm{p,vpi}}\right)_x = (F - q_{\mathrm{sh}}h_{\mathrm{e}})\left(\frac{\varphi_{\mathrm{v}}}{m+1}\right)x \tag{9}$$

It can be seen from Eqs. (8) and (9) that the internal moments of the RC beam and steel plate components in the shear span distant x from the support are dependent on the degree of transverse partial interaction φ_{v} and the centroidal axis separation h_{e}.

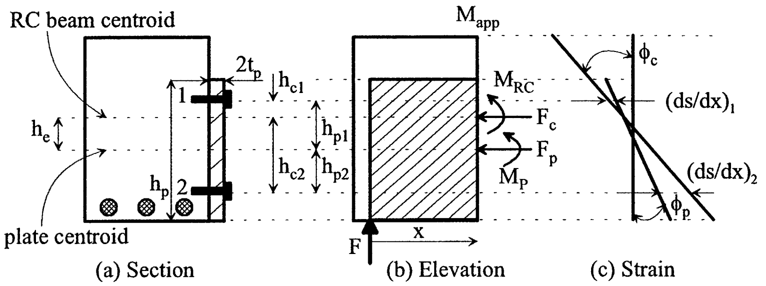

Fig. 4. Mixed analysis of side-plated beam.

4. Longitudinal deformations due to both longitudinal and transverse partial interaction

4.1. Longitudinal slip strains

Fig. 4 shows a side-plated beam with two rows of connectors, where the numbers 1 and 2 denote the first and second rows of shear connectors from the top. The equivalent section with a plate of thickness $2t_p$ is shown in Fig. 4(a), in which the distances from the bolt levels to the centroid of the RC component are h_{c1} and h_{c2} and those to the centroid of the plate component are h_{p1} and h_{p2} respectively. As before, the distance between the centroids of the concrete and steel components is h_e. In Fig. 4(c), the longitudinal slip strains at the bolt levels 1 and 2 are denoted $(ds_L/dx)_1$ and $(ds_L/dx)_2$ respectively.

Based on the definition of slip strains, and taking due account of the actions acting on the shear span x from the support (Fig. 4(b)), the equations for the slip strain at the two bolt levels can be stated as

$$\left(\frac{ds_L}{dx}\right)_1 = \varepsilon_{c1} - \varepsilon_{p1} = \left(\frac{-M_{RC}h_{c1}}{(EI)_{RC}} - \frac{F_c}{(AE)_{RC}}\right) - \left(\frac{-M_p h_{p1}}{(EI)_p} + \frac{F_p}{(AE)_p}\right) \tag{10a}$$

$$\left(\frac{ds_L}{dx}\right)_2 = \varepsilon_{c2} - \varepsilon_{p2} = \left(\frac{M_{RC}h_{c2}}{(EI)_{RC}} - \frac{F_c}{(AE)_{RC}}\right) - \left(\frac{-M_p h_{p2}}{(EI)_p} + \frac{F_p}{(AE)_p}\right) \tag{10b}$$

Noting from Fig. 4(a) that $h_{p1} - h_{c1} = h_{c2} - h_{p2} = h_e$ and that $M_{RC}/(EI)_{RC} = M_p/(EI)_p + \Delta\phi$, Eqs. (10a) and (10b) can be further simplified as

$$\left(\frac{ds_L}{dx}\right)_1 = \left(\frac{M_p h_e}{(EI)_p} - P_{sh}\sum(\overline{A}\,\overline{E})\right) - \Delta\phi h_{c1} \tag{11a}$$

$$\left(\frac{ds_L}{dx}\right)_2 = \left(\frac{M_p h_e}{(EI)_p} - P_{sh}\sum(\overline{A}\,\overline{E})\right) + \Delta\varphi h_{c2} \tag{11b}$$

where $P_{sh} = F_c = F_p$ and $\sum(\overline{A}\,\overline{E}) = 1/(AE)_{RC} + 1/(AE)_p$. Substituting M_p from Eq. (4) into Eq. (11) and rearranging yields

$$\left(\frac{ds_L}{dx}\right)_1 = M_{app}\frac{h_e}{\sum(EI)} - P_{sh}\left(\frac{h_e^2}{\sum(EI)} + \sum(\overline{A}\,\overline{E})\right) - \Delta\phi\left(\frac{h_e(EI)_{RC}}{\sum(EI)} + h_{c1}\right) \tag{12a}$$

$$\left(\frac{ds_L}{dx}\right)_2 = M_{app}\frac{h_e}{\sum(EI)} - P_{sh}\left(\frac{h_e^2}{\sum(EI)} + \sum(\overline{A}\,\overline{E})\right) - \Delta\phi\left(\frac{h_e(EI)_{RC}}{\sum(EI)} - h_{c2}\right) \tag{12b}$$

where $\sum(EI) = (EI)_{RC} + (EI)_p$.

By setting $K_1 = h_e/\sum(EI)$ and $K_2 = h_e^2/(EI) + \sum(\overline{A}\overline{E})$, Eq. (12) can be rewritten as

$$\left(\frac{ds_L}{dx}\right)_1 = M_{app}K_1 - P_{sh}K_2 + \Delta\phi\left(-h_{c1} - K_1(EI)_{RC}\right) \tag{13a}$$

$$\left(\frac{ds_L}{dx}\right)_2 = M_{app}K_1 - P_{sh}K_2 + \Delta\phi\left(h_{c2} - K_1(EI)_{RC}\right) \tag{13b}$$

It can be seen from Eq. (13) that when there is transverse full interaction, i.e. $\Delta\phi = 0$, then

$$\left(\frac{ds_L}{dx}\right)_{1,vfi} = \left(\frac{ds_L}{dx}\right)_{2,vfi} = M_{app}K_1 - P_{sh}K_2.$$

Substituting this relationship into Eq. (13) produces

$$\left(\frac{ds_L}{dx}\right)_{1,vpi} = \left(\frac{ds_L}{dx}\right)_{1,vfi} + \Delta\phi\left[-h_{c1} - K_1(EI)_{RC}\right] \tag{14a}$$

$$\left(\frac{ds_L}{dx}\right)_{2,vpi} = \left(\frac{ds_L}{dx}\right)_{2,vfi} + \Delta\phi\left[+h_{c2} - K_1(EI)_{RC}\right] \tag{14b}$$

where the subscript vfi denotes vertical (or transverse) full interaction. It can be concluded from Eq. (14) that the slip strain at a certain shear connector level under a condition of transverse partial interaction is directly proportional to that due to transverse full interaction, and the difference between the RC and plate curvatures $\Delta\phi$. Note that in the preceding equations the sign of the distances h_{c1} and h_{c2} depends on the location of the bolts. The sign is thus negative when the bolt is located in a compressive strain region, and positive when it is located in a tensile strain region.

4.2. Longitudinal slips

If only one line of shear connectors placed along the centroidal axis of the plate was used in the plating procedure then $h_{c1} = h_e$, and Eq. (13a) then becomes

$$\frac{ds_L}{dx} = M_{app}K_1 - P_{sh}K_2 - \Delta\phi\left[(EI)_{RC}K_1 - h_e\right] \tag{15a}$$

or

$$\frac{ds_L}{dx} = (FK_1 - q_{sh}K_2)x - \frac{\Delta\phi_{max}}{L}x\left[(EI)_{RC}K_1 - h_e\right] \tag{15b}$$

Substituting Eq. (7) into Eq. (15b), simplifying and noting that $K_1 = h_e/\sum(EI)$ gives

$$\frac{ds_L}{dx} = \left\{(FK_1 - q_{sh}K_2) + \frac{(1-\varphi_v)(F - q_{sh}h_e)K_1}{m}\right\}x \tag{16}$$

Integrating Eq. (16) produces the longitudinal slip distribution along the shear span x as

$$s_L(x) = \left\{(FK_1 - q_{sh}K_2) + \frac{(1-\varphi_v)(F - q_{sh}h_e)K_1}{m}\right\}\frac{(L^2 - x^2)}{2} \tag{17}$$

It can be seen from Eq. (17) that the longitudinal slip along the shear span x varies parabolically with a maximum value at the support and a value of zero at midspan (when $x = L$) owing to the symmetry of the beam and its loading.

4.3. Relationship between maximum longitudinal and transverse slips

The maximum longitudinal slip at the support can be obtained by substituting $x = 0$ in Eq. (17). Hence,

$$(s_{\mathrm{L,max}})_{\mathrm{vpi}} = (M_{\mathrm{app,max}}K_1 - P_{\mathrm{sh,max}}K_2)\frac{L}{2} + \frac{(1-\varphi_{\mathrm{v}})(F - q_{\mathrm{sh}}h_{\mathrm{e}})K_1L^2}{2m} \tag{18}$$

Clearly from Eq. (18) for transverse full interaction when $\varphi_{\mathrm{v}} = 1$,

$$(s_{\mathrm{L,max}})_{\mathrm{vfi}} = \left(M_{\mathrm{app,max}}K_1 - P_{\mathrm{sh,max}}K_2\right)\frac{L}{2} \tag{19}$$

and noting further that

$$s_{\mathrm{v,max}} = \frac{1}{3(m+1)}L^3(1-\varphi_{\mathrm{v}})(F - q_{\mathrm{sh}}h_{\mathrm{e}})\sum\overline{EI} \tag{20}$$

then Eq. (18) can be simplified to yield the expression

$$(s_{\mathrm{L,max}})_{\mathrm{vpi}} = (s_{\mathrm{L,max}})_{\mathrm{vfi}} + \frac{3h_{\mathrm{e}}}{2L(m+1)}s_{\mathrm{v,max}} \tag{21}$$

Furthermore, the maximum longitudinal slip for transverse full interaction can be expressed in terms of the degree of longitudinal partial interaction φ_{L} as

$$(s_{\mathrm{L,max}})_{\mathrm{vfi}} = \frac{M_{\mathrm{app,max}}K_1L}{2}(1-\varphi_{\mathrm{L}}) \tag{22}$$

where φ_{L} is defined as the ratio of the longitudinal shear forces exerted by the shear connectors in a shear span to the longitudinal shear force required so that there is full longitudinal interaction at mid-span, that is the slip strain at mid-span is zero (Oehlers et al., 1997). It can now be concluded from Eqs. (20)–(22) that the maximum longitudinal slip when there is transverse partial interaction depends on the degrees of both longitudinal and partial interaction.

5. Neutral axis separation

Fig. 5 shows the strain profiles in the RC beam and plate ε_{c} and ε_{p} respectively. These strain profiles intersect the vertical *y–y* axis at two points, A and E, and at the level of the bolts, B is the point on the plate strain profile and C is the point on the RC beam strain profile. These points would clearly indicate that there is a longitudinal slip strain due to transverse partial interaction BC $= (\mathrm{d}s_{\mathrm{L}}/\mathrm{d}x)_{\mathrm{vpi}}$ and as a result of the neutral axis separation $h_{\mathrm{na,vpi}}$. In this section, we shall quantify this neutral axis separation of the strain profiles in terms of the other parameters determined previously.

Let us draw a line from the point B parallel to the strain profile of the RC component which cuts the vertical *y–y* axis at D, and we will call the distance AD $h^*_{\mathrm{na,vpi}}$ as shown in Fig. 5. When the difference between the component curvatures $\Delta\phi = \phi_{\mathrm{c}} - \phi_{\mathrm{s}}$ is relatively small in comparison to the individual component curvatures, then D $\to$ E and $h^*_{\mathrm{na,vpi}} \to h_{\mathrm{na,vpi}}$. Thus if this condition is valid then $h^*_{\mathrm{na,vpi}} \approx h_{\mathrm{na,vpi}}$. From Fig. 5, the relationship between the longitudinal slip strain and the neutral axis separation can be written as

$$h_{\mathrm{na,vpi}} \approx h^*_{\mathrm{na,vpi}} = \frac{(\mathrm{d}s_{\mathrm{L}}/\mathrm{d}x)_{\mathrm{vpi}}}{\phi_{\mathrm{c}}} \tag{23}$$

If the value of the slip strain for the side-plated beam with a single line of bolts given in Eq. (15a) is substituted into Eq. (23), and it is noted that $(EI)_{\mathrm{c}}K_1 - h_{\mathrm{e}} = -h_{\mathrm{e}}/(m+1)$, then

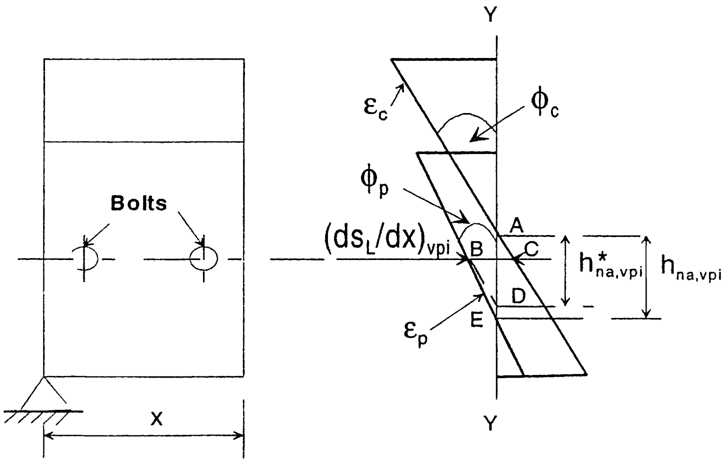

Fig. 5. Neutral axis separation of the concrete and plate strain profiles.

$$h_{\mathrm{na,vpi}} = \frac{M_{\mathrm{app}}K_1 - P_{\mathrm{sh}}K_2}{\phi_{\mathrm{c}}} + \frac{\Delta\phi h_{\mathrm{e}}}{\phi_{\mathrm{c}}(m+1)} \tag{24}$$

As the value of the internal moments varies linearly with x as a result of the linear elastic properties and the point load at midspan, the difference between the element curvatures $\Delta\phi$ is a linear function of the shear span, and if $\Delta\phi_{\mathrm{max}}$ is the maximum curvature which occurs at $x = L$ and noting $\Delta\phi = 0$ at $x = 0$ then

$$\frac{\Delta\phi}{\phi_{\mathrm{c}}} = \frac{\Delta\phi_{\mathrm{max}}}{\phi_{\mathrm{c,max}}} \tag{25}$$

Substituting Eq. (25) into Eq. (24) and making use of Eq. (7) and the relationship $(\mathrm{d}s_{\mathrm{L}}/\mathrm{d}x)_{\mathrm{vfi}} = M_{\mathrm{app}}K_1 - P_{\mathrm{sh}}K_2$ produces

$$h_{\mathrm{na,vpi}} = \frac{(\mathrm{d}s_{\mathrm{L}}/\mathrm{d}x)_{\mathrm{vfi}}}{\phi_{\mathrm{c}}} + \frac{(1-\varphi_{\mathrm{v}})(F - q_{\mathrm{sh}}h_{\mathrm{e}})Lh_{\mathrm{e}}}{\phi_{\mathrm{c,max}}(EI)_{\mathrm{RC}}(m+1)} \tag{26}$$

The maximum internal moment in the RC component can be obtained from Eq. (8) by substituting $x = L$ as

$$(M_{\mathrm{RC,max}})_{\mathrm{vpi}} = (M_{\mathrm{RC,L}})_{\mathrm{vpi}} = (F - q_{\mathrm{sh}}h_{\mathrm{e}})\left(1 - \frac{\varphi_{\mathrm{v}}}{m+1}\right)L \tag{27}$$

and since $\phi_{\mathrm{c,max}}(EI)_{\mathrm{RC}} = (M_{\mathrm{RC,max}})_{\mathrm{vpi}}$ and $h_{\mathrm{na,vfi}} = (\mathrm{d}s_{\mathrm{L}}/\mathrm{d}x)_{\mathrm{vfi}}/\phi_{\mathrm{c}}$, Eq. (26) can be further simplified using Eq. (27) as

$$h_{\mathrm{na,vpi}} = h_{\mathrm{na,vfi}} + \frac{h_{\mathrm{e}}(1-\varphi_{\mathrm{v}})}{m + (1-\varphi_{\mathrm{v}})} \tag{28}$$

In addition, the authors (Nguyen et al., 1998) showed that

$$h_{\mathrm{na,vfi}} = \frac{h_{\mathrm{e}}K_3(1-\varphi_{\mathrm{L}})}{K_3 - \varphi_{\mathrm{L}}} \tag{29}$$

where $K_3 = K_2/(K_1h_{\mathrm{e}})$. Using this, Eq. (28) can be written the following dimensionless form.

$$\frac{h_{\mathrm{na,vpi}}}{h_{\mathrm{e}}} = \frac{K_3(1-\varphi_{\mathrm{L}})}{K_3 - \varphi_{\mathrm{L}}} + \frac{(1-\varphi_{\mathrm{v}})}{m + (1-\varphi_{\mathrm{v}})} \tag{30}$$

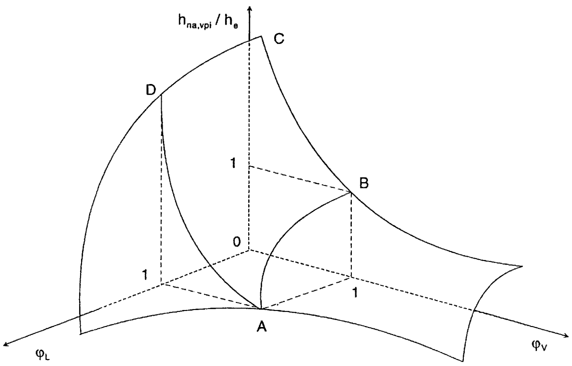

Fig. 6. A 3-D picture of the normalised neutral axis separation vs degrees of longitudinal and transverse partial interaction.

Fig. 6 shows a three-dimensional representation of the normalised neutral axis separation of a beam subjected to various degrees of both longitudinal and transverse partial interaction. It can be seen from this figure that only the surface ABCD is significant, since it represents the real situation of a side-plated RC beam where both degrees of partial interaction ϕ_L and ϕ_v vary from 0 to 1.

6. Concluding remarks

A technique is currently being developed for bolting plates to the sides of existing RC beams in order to both strengthen and stiffen them. In this form of composite construction, the shear connection is subjected not only to longitudinal partial interaction, but also to transverse partial interaction. An analysis technique is being developed which allows the longitudinal shear forces to be resisted by the plastic capacity of the shear connectors. However, the technique hitherto has been based on the elastic stiffness of the shear connectors to resist the transverse partial interaction, in order to restrict the difference in the curvatures between the steel and concrete components.

Mathematical models for transverse partial interaction and its interaction with longitudinal partial interaction have been developed in this paper. These models have been developed in a form that will eventually lead to design rules for this relatively new system of structural rehabilitation, and which require experimental validation.

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